Master 2 internship
Parallel solvers for anisotropic problems issued from magnetized plasma physics

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This internship takes place in a research project aimed at introducing innovative numerical methods for the simulation of magnetized plasmas. The applications targeted are related to either thermonuclear fusion (ITER project[1]) or electric propulsion[2], used to operate constellations of nano and micro satellites.

The key point here lies in the confinement of ionized gazes (plasma) thanks to an intense magnetization, binding the motion of the charged particles along the magnetic field lines. On macroscopic scales, reflecting the collective properties of the particles, the equations describing the plasma evolution are severely anisotropic. This may be modeled thanks to the toy problem

$$\begin{cases} -\Delta_\perp \phi^\varepsilon - \dfrac{1}{\varepsilon}\Delta_\parallel \phi^\varepsilon = f^\varepsilon \,, & \text{in } \Omega\,, \\ \mathbf{n}\cdot\big(\nabla_\perp \phi^\varepsilon + \dfrac{1}{\varepsilon}\nabla_\perp \phi^\varepsilon\big) = 0\,, & \text{on } \Gamma_N\,, \\ \phi^\varepsilon = 0\,, & \text{on } \Gamma_D\,. \end{cases} \tag{1a}$$

where $\mathbf{n}$ denotes the outward normal to the domain Ω, $\partial\Omega = \Gamma_N \cup \Gamma_D$ is the domain boundary, with $\mathbf{b}\cdot\mathbf{n} = 0$ on Γ_D, $\mathbf{b}$ denoting the unit vector pointing in the anisotropy direction and

$$\nabla_\parallel \psi = (\mathbf{b}\otimes\mathbf{b})\psi\,, \qquad \nabla_\perp \psi = (\mathbb{I}\mathrm{d} - \mathbf{b}\otimes\mathbf{b})\psi\,, \qquad \Delta_{\parallel,\perp}\psi = \nabla\cdot(\nabla_{\parallel,\perp}\psi)\,. \tag{1b}$$

[1]International Thermonuclear Experimental Reactor https://www.iter.org/proj/inafewlines

[2]https://www.esa.int/Enabling_Support/Space_Engineering_Technology/What_is_Electric_propulsion: "Electric propulsion is currently considered by all space actors as a key and revolutionary technology for the new generations of commercial and scientific satellites. Initiatives in this field all over the world are aimed at the development of competitive new generations of EPS. In Europe too, all stakeholders including the European Space Agency (ESA), the National Space Agencies and industrial players have been setting efforts to develop and increase the competitiveness of the European EP technology for the different types of markets since a few decades ago." (EP(S): Electric Propulsion (Systems))

The anisotropy strength is ε^{-1}, $\mathbb{I}\mathrm{d}$ being the identity matrix and $\otimes$ the tensor product. The solutions of this class of problems develop hardly no gradients along the magnetic field lines (when $\varepsilon \ll 1$) while the variations in the perpendicular directions remain significant, as illustrated by the plots of Fig. 1.

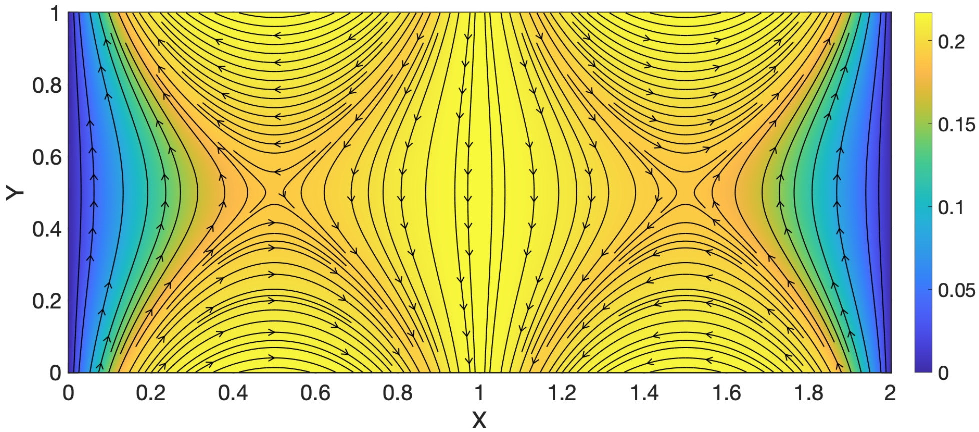


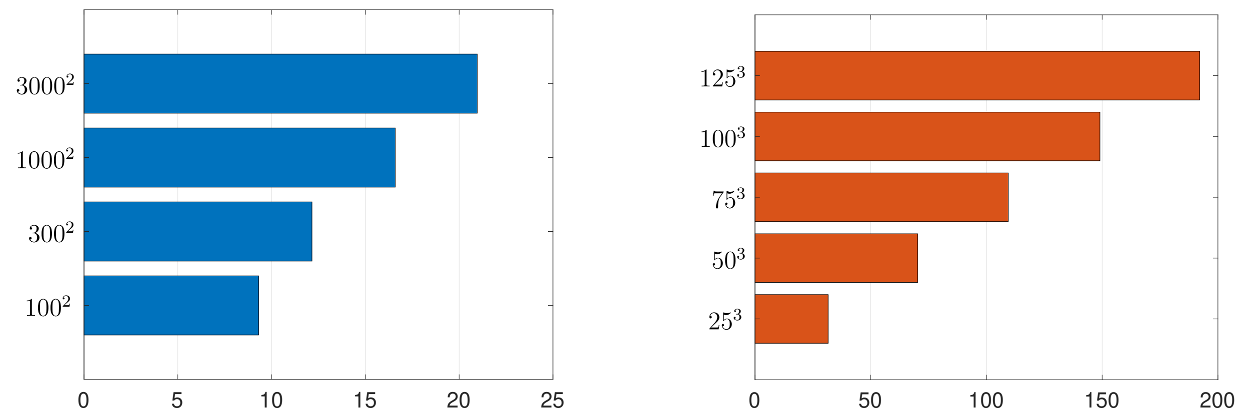
Figure 1: Magnetic field lines (plasma thruster with cusps) and solution iso-values of the associated anisotropic problem as defined by Eqs. (1).

This property is difficult to obtain from a numerical scheme. This is a bottleneck in the derivation of efficient and reliable numerical methods, the simulation of magnetized plasma being at the core of major scientific challenges. A loss of precision of discrete approximations are indeed observed when the mesh is unrelated to the magnetic field geometry [1]. The condition number of the linear systems issued from the discretization of these anisotropic equations is also increasing with the anisotropy (magnetization) strength [2]. The strategies classically implemented consists in using high order schemes together with preconditioners in order to alleviate the bad conditioning of the linear system. These attempts reveal unsuccessful to address the tremendous anisotropies of the targeted applications.

For a decade, the Institute of Mathematics of Toulouse has developed numerical methods [1, 2, 3] giving rise to discrete approximations with a precision unaltered by the anisotropy strength and carried out on meshes unrelated to the magnetic field. These methods rely on a reformulation of the problem thanks to the introduction of an auxiliary variable providing an equivalent set of equations in which the anisotropy has vanished. So far, the linear systems stemming from the discretization of these reformulated equations have been solved by sparse direct solvers. Though these tools are very efficient for two dimensional computations, their numerical costs are quite expensive for three dimensional computations as reported on Fig. 2.

The purpose of the present work is to improve the efficiency of solvers dedicated to linear systems issued for the reformulated (Micro-Macro) equations and investigate alternative approaches to sparse direct solvers. Specifically, the work consists in developing a discretization of the model problem on a regular grid with a domain decomposition strategy, then in

1. optimizing the efficiency of parallel sparse direct solvers;



(a) Two dimensional problem.

(b) Three dimensional problem.

Figure 2: Number of non zeros elements in the factorized matrix related to the number of non zeros element in the matrix issued from the Micro-Macro method [3].

2. developing hybrid solvers implementing block factorizations together with iterative methods;

3. investigating the efficiency of adaptive domain decomposition methods [4];

4. assessing the effectiveness of (algebraic and geometric) multi-grid methods.

The internship will take place within a collaboration between IMT (Institut de Mathématiques de Toulouse), IRIT (Institut de Recherche en Informatique de Toulouse). The candidate should have a strong interest for applied mathematics, computational sciences and high-performance computing. A background on scientific libraries [5] would be appreciated.

The work initiated during this internship may be continued in the frame of a PhD thesis.

References

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