Singular perturbated problems and julia package in optimal control.

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Entreprise / Laboratoire : CIMI & INP-ENSEEIHT-IRIT

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Contexte. We have two objectives in this project. First, we are interested to study and solve singular perturbated optimal control problems and in particular, optimal control problems with turnpike properties [8, 9] which are actually study several research teams. The difficulties for solving such problems come from the fact that the two-point boundary value problems (TPBVP) obtained from the application of the PMP is a singular perturbated one. In our knowledge, the best methods for solving singular perturbated boundary value problems are based on the discretization of the TPBVP with a refinement of the discretized grid [6, 10, 11]. This is, in fact, in relation with the direct methods, because for regular problems and suitable symplectic numerical integration the scheme of figure 1 commutes.

$$\begin{tikzcd}
\text{Optimal Control Problem} \arrow[r, "\text{Necessary Condition (PMP)}"] \arrow[d, "\text{Discretization}"'] & \text{Two-Point Boundary Value Problem} \arrow[d, "\text{Discretization}"] \\
\text{Optimization Problem in finite dimension} \arrow[r, "\text{Necessary Condition (KKT)}"] & \text{Non linear Equation}
\end{tikzcd}$$

Figure 1. *Commutative scheme when we use suitable symplectic numerical integration.*

Our conviction is that for solving numerically singular perturbated optimal control problems, with a good precision, it is necessary to use a good connection between direct, indirect and homotopy methods. In particular, inside the path-following routine, by analyzing the solution for a fixed value of the homotopic parameter, we can determine the good discretization which would be necessary to avoid the numerical difficulties we have to compute this path.

This last point is the objective of the ADT project of INRIA, named Control Toolbox (CT), which realizes the interoperability of the bocop, based on direct methods, and `HamPath`, based on indirect and homotopy methods, softwares, with an interface to the Python language. The realization of this project can be seen on the website `https://ct.gitlabpages.inria.fr/gallery`. But the actual version is limited because the implementation of automatic differentiation (which is a central tool used in bocop and in `HamPath`) in Python is not yet reliable and the performance of the Python language is not good compared to a compiled langage or to julia. Therefore, the second objective of the project is to develop a package in julia, which will be incorporated into the ADT project, for solving optimal control problems that contains :
- Direct methods
- Indirect methods
- Path-following methods
- Differed correction method for nonlinear two-point boundary value problems

With this julia Package, we could develop specific routines for singular perturbated optimal control problems with the integration of grid discretization refinement inside the path-following routine.

The other point that this tool could help us with is the study of the link between the existence of what we call, in optimal control theory, singular arcs and singular perturbations in optimal control problems.

Travail à réaliser. Dans le cadre de ce stage le stagiaire travaillera sous l'environnement gitlab . Il sera amener à :
• Réaliser sur des exemples simples des tests numériques en julia et Matlab de résolution de problèmes de contrôle optimal avec perturbations singulières.
• développer en julia une interface au code `HamPath`[13] développé dans l'équipe APO de l'IRIT.
• Intégrer le travail dans le cadre du projet : https://github.com/control-toolbox/ControlToolbox.jl

Compétences / Connaissances minimales requises.
• De bonnes bases en mathématiques appliquées.
• De bonnes bases en programmation, le développement logiciel sera réalisé dans l'environnement git avec la génération de la documentation, les tests unitaires et l'intégration continue
• Des bases en Matlab et ou Julia.

Perspectives. Ce sujet pourra être poursuivi en thèse. Pour cela le candidat devra candidater aux bourse de thèse de CIMI et de l'IRIT.

Références

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