Least Squares Probem

Example : $C^{14} Datation

Radioactive carbon ^{14}C is produced in the atmosphere by the effect of cosmic rays on atmospheric nitrogen. It is oxidized to $^{14}CO_2$ and absorbed in this form by living organisms. So, living organisms contain a certain percentage of radioactive carbon relative to ^{12}C and ^{13}C which are stable. We suppose that carbon production ^{14}C is constant over the last few millennia.

It is also assumed that, when an organism dies, its exchanges with the atmosphere cease, and that radioactivity due to carbon to carbon ^{14}C decreases according to the following exponential law:

$$A(t, A_0, \lambda) = A_0 e^{-\lambda t}.$$

The analysis of the trunks (wood is dead tissue) of old trees {} and {} furnishes us~:

- its age t in year
- its radioactivity A

t_i	500	1000	2000	3000	4000	5000	6300
A_i	14.5	13.5	12.0	10.8	9.9	8.9	8.0

We want to find the values of the parameters A_0 and λ, so that the function $A(t, A_0, \lambda)$ is “near” the data :

$$(P) \begin{cases} Min\ f(\beta) = \frac{1}{2}\|r(\beta\|^2 = f(A_0, \lambda) = \frac{1}{2}\sum_{i=1}^{n}(A_i - A_0 e^{-\lambda t_i})^2 \\ \beta = (A_0, \lambda) \in \mathbf{R}^2. \end{cases}$$

Solve this problem by using :

- The Newton algorithm
- The Gauss-Newton algorithm :

$$(P_k) \begin{cases} Min\ \ f_k(s) = \frac{1}{2}\|r(\beta^{(k)}) + J_r(\beta^{(k)})s\|^2 \\ s \in \mathbf{R}^p, \end{cases}$$

where $s = \beta - \beta^{(k)}$ abd $J_r(\beta)$ is the Jacobian matrix of r in β.